3-2 Videos Guide

3-2a

Theorem (statement):

- Rolle's Theorem: Let f be a function such that:
  1. f is continuous on $[a, b]$.
  2. f is differentiable on (a, b).
  3. $f(a) = f(b)$.

  Then there is a number $c \in (a, b)$ such that $f'(c) = 0$.

Exercises:

- Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.
  $f(x) = x^3 - 2x^2 - 4x + 2, \quad [-2, 2]$
- Let $f(x) = \tan x$. Show that $f(0) = f(\pi)$ but there is no number c in $(0, \pi)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

3-2b

Proof:

- Rolle's Theorem

Theorem (statement and proof):

- Mean Value Theorem: Let f be a function such that:
  1. f is continuous on $[a, b]$.
  2. f is differentiable on (a, b).

  Then there is a number $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
- Mathematical theory (how math works)

3-2c

Exercises:

- Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.
  $f(x) = \frac{1}{x}, \quad [1, 3]$
- Show that the equation has exactly one real root.
  $2x - 1 - \sin x = 0$